M11/5/MATHL/HP1/ENG/TZ2/XX/M

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MARKSCHEME

May 2011

MATHEMATICS

Higher Level

Paper 1

20 pages

https://xtremepape.rs/

*This markscheme is **confidential** and for the exclusive use of examiners in this examination session.*

*It is the property of the International Baccalaureate and must **not** be reproduced or distributed to any other person without the authorization of IB Cardiff.*

https://xtremepape.rs/

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Write the marks in red on candidates' scripts, in the right hand margin.

- Show the **breakdown** of individual marks awarded using the abbreviations ***M1***, ***A1***, *etc*.
- Write down the total for each **question** (at the end of the question) and **circle** it.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

https://xtremepape.rs/

4 Implied marks

Implied marks appear in ***brackets e.g. (M1)****, and can only be awarded if* ***correct*** *work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (****FT****) marks are awarded where an incorrect answer from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s). To award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (****MR****). Apply a* ***MR*** *penalty of 1 mark to that question. Award the marks as usual and then write –1(****MR****) next to the total. Subtract 1 mark from the total for the question. A candidate should be penalized only once for a particular mis-read.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. The mark should be labelled ***(d)*** *and a brief* ***note*** *written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

https://xtremepape.rs/

9 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

  Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

  $$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right)$$ ***A1***

  Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Candidates should be penalized ***once only IN THE PAPER*** *for an accuracy error* ***(AP)***. *Award the marks as usual then write* ***(AP)*** *against the answer. On the* ***front*** *cover write –1****(AP)***. *Deduct 1 mark from the total for the paper, not the question.*

- If a final correct answer is incorrectly rounded, apply the ***AP***.
- If the level of accuracy is not specified in the question, apply the ***AP*** for correct answers not given to three significant figures.

If there is no working shown, and answers are given to the correct two significant figures, apply the ***AP***. However, do **not** accept answers to one significant figure without working.

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

https://xtremepape.rs/

SECTION A

1. (a) **METHOD 1**

$f'(x) = q - 2x = 0$	***M1***
$f'(3) = q - 6 = 0$	
$q = 6$	***A1***
$f(3) = p + 18 - 9 = 5$	***M1***
$p = -4$	***A1***

METHOD 2

$f(x) = -(x-3)^2 + 5$	***M1A1***
$\quad = -x^2 + 6x - 4$	
$q = 6,\ p = -4$	***A1A1***

(b) $g(x) = -4 + 6(x-3) - (x-3)^2\ (= -31 + 12x - x^2)$ ***M1A1***

> **Note:** Accept any alternative form which is correct.
> Award ***M1A0*** for a substitution of $(x+3)$.

[6 marks]

2. (a) $A^2 = \begin{pmatrix} 2a & -2 \\ -a & 2a+1 \end{pmatrix}$ ***(M1)A1***

(b) **METHOD 1**

$\det A^2 = 4a^2 + 2a - 2a = 4a^2$	***M1***	
$a = \pm 2$	***A1A1***	***N2***

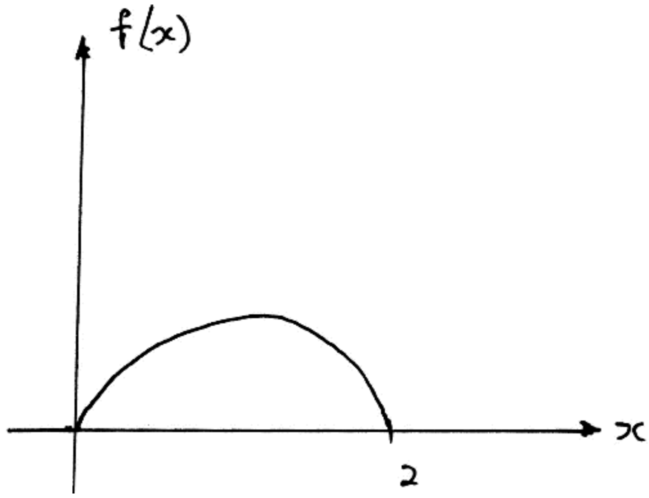
METHOD 2

$\det A = -2a$	***M1***	
$\det A = \pm 4$		
$a = \pm 2$	***A1A1***	***N2***

[5 marks]

https://xtremepape.rs/

3. (a)



A1

Note: Award ***A1*** for intercepts of 0 and 2 and a concave down curve in the given domain .

Note: Award ***A0*** if the cubic graph is extended outside the domain $[0, 2]$.

(b) $\int_0^2 kx(x+1)(2-x)\,dx = 1$ ***(M1)***

Note: The correct limits and =1 must be seen but may be seen later.

$k\int_0^2 (-x^3 + x^2 + 2x)\,dx = 1$ ***A1***

$k\left[-\frac{1}{4}x^4 + \frac{1}{3}x^3 + x^2\right]_0^2 = 1$ ***M1***

$k\left(-4 + \frac{8}{3} + 4\right) = 1$ ***(A1)***

$k = \frac{3}{8}$ ***A1***

[6 marks]

https://xtremepape.rs/

4. (a) $\text{AB} = \sqrt{1^2 + (2 - \sqrt{3})^2}$ ***M1***

$= \sqrt{8 - 4\sqrt{3}}$ ***A1***

$= 2\sqrt{2 - \sqrt{3}}$ ***A1***

(b) **METHOD 1**

$\arg z_1 = -\frac{\pi}{4}$ $\arg z_2 = -\frac{\pi}{3}$ ***A1A1***

Note: Allow $\frac{\pi}{4}$ and $\frac{\pi}{3}$.

Note: Allow degrees at this stage.

$\text{A}\hat{\text{O}}\text{B} = \frac{\pi}{3} - \frac{\pi}{4}$

$= \frac{\pi}{12}$ (accept $-\frac{\pi}{12}$) ***A1***

Note: Allow ***FT*** for final ***A1***.

METHOD 2

attempt to use scalar product or cosine rule ***M1***

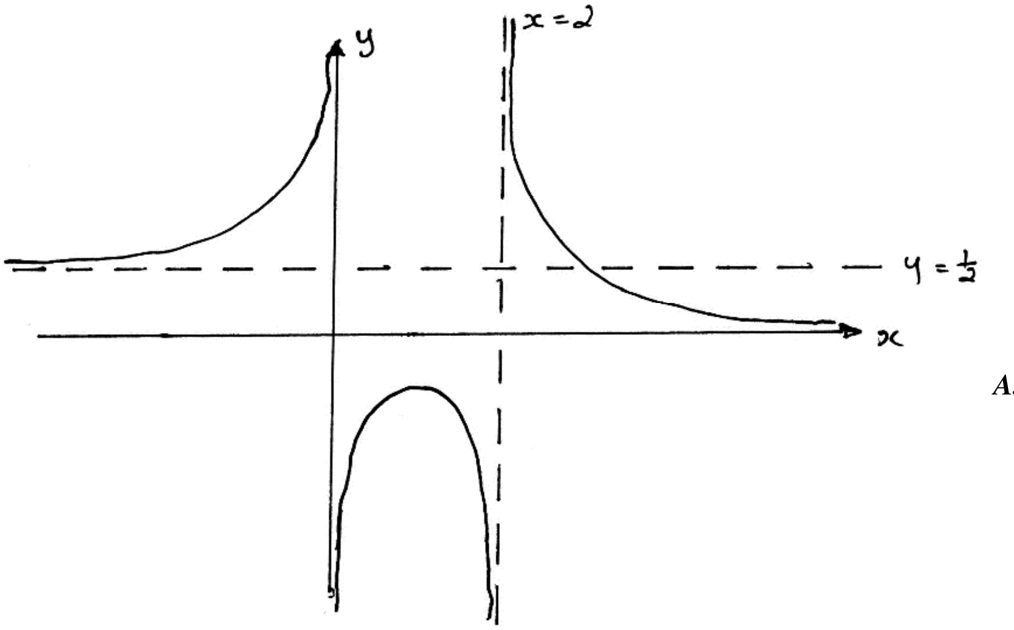
$\cos \text{A}\hat{\text{O}}\text{B} = \frac{1+\sqrt{3}}{2\sqrt{2}}$ ***A1***

$\text{A}\hat{\text{O}}\text{B} = \frac{\pi}{12}$ ***A1***

[6 marks]

https://xtremepape.rs/

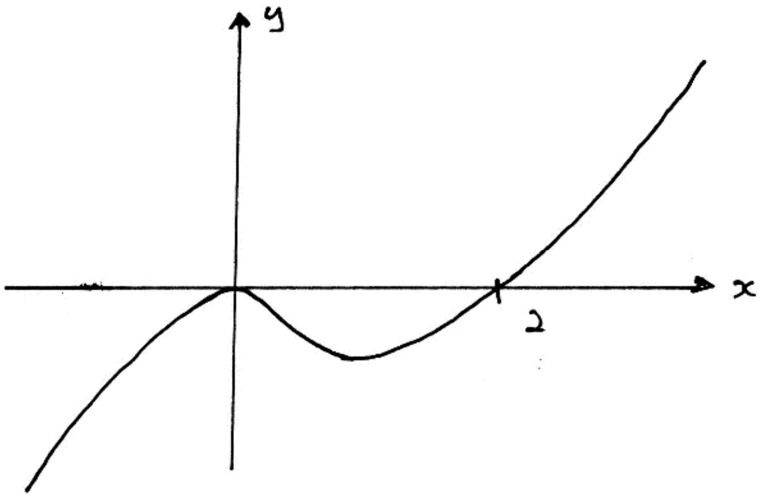
5. (a)



A3

Note:	Award ***A1*** for each correct branch with position of asymptotes clearly indicated. If $x = 2$ is not indicated, only penalise once.

(b)



A3

Note:	Award ***A1*** for behaviour at $x = 0$, ***A1*** for intercept at $x = 2$, ***A1*** for behaviour for large $\lvert x \rvert$.

[6 marks]

https://xtremepape.rs/

6. (a) $\overrightarrow{\text{CB}} = \boldsymbol{b} - \boldsymbol{c}$, $\overrightarrow{\text{AC}} = \boldsymbol{b} + \boldsymbol{c}$ ***A1A1***

Note: Condone absence of vector notation in (a).

(b) $\overrightarrow{\text{AC}} \bullet \overrightarrow{\text{CB}} = (\boldsymbol{b} + \boldsymbol{c}) \bullet (\boldsymbol{b} - \boldsymbol{c})$ ***M1***

$= |\boldsymbol{b}|^2 - |\mathbf{c}|^2$ ***A1***

$= 0$ since $|\boldsymbol{b}| = |\mathbf{c}|$ ***R1***

Note: Only award the ***A1*** and ***R1*** if working indicates that they understand that they are working with vectors.

so $\overrightarrow{\text{AC}}$ is perpendicular to $\overrightarrow{\text{CB}}$ *i.e.* $\text{A}\hat{\text{C}}\text{B}$ is a right angle ***AG***

[5 marks]

7. (a) area of AOP $= \frac{1}{2} r^2 \sin\theta$ ***A1***

(b) $\text{TP} = r\tan\theta$ ***(M1)***

area of POT $= \frac{1}{2} r (r \tan\theta)$

$= \frac{1}{2} r^2 \tan\theta$ ***A1***

(c) area of sector OAP $= \frac{1}{2} r^2 \theta$ ***A1***

area of triangle OAP < area of sector OAP < area of triangle POT ***R1***

$\frac{1}{2} r^2 \sin\theta < \frac{1}{2} r^2 \theta < \frac{1}{2} r^2 \tan\theta$

$\sin\theta < \theta < \tan\theta$ ***AG***

[5 marks]

https://xtremepape.rs/

8. $x = 2e^y - \frac{1}{e^y}$ ***M1***

Note: The ***M1*** is for switching the variables and may be awarded at any stage in the process and is awarded independently. Further marks do not rely on this mark being gained.

$xe^y = 2e^{2y} - 1$

$2e^{2y} - xe^y - 1 = 0$ ***A1***

$e^y = \frac{x \pm \sqrt{x^2 + 8}}{4}$ ***M1A1***

$y = \ln\left(\frac{x \pm \sqrt{x^2 + 8}}{4}\right)$

therefore $h^{-1}(x) = \ln\left(\frac{x + \sqrt{x^2 + 8}}{4}\right)$ ***A1***

since ln is undefined for the second solution ***R1***

Note: Accept $y = \ln\left(\frac{x + \sqrt{x^2 + 8}}{4}\right)$.

Note: The ***R1*** may be gained by an appropriate comment earlier.

[6 marks]

https://xtremepape.rs/

9. (a) **METHOD 1**

$$\mathrm{P}(3 \text{ defective in first } 8) = \binom{8}{3} \times \frac{4}{15} \times \frac{3}{14} \times \frac{2}{13} \times \frac{11}{12} \times \frac{10}{11} \times \frac{9}{10} \times \frac{8}{9} \times \frac{7}{8}$$ ***M1A1A1***

Note:	Award ***M1*** for multiplication of probabilities with decreasing denominators. Award ***A1*** for multiplication of correct eight probabilities. Award ***A1*** for multiplying by $\binom{8}{3}$.

$$= \frac{56}{195}$$ ***A1***

METHOD 2

$$\mathrm{P}(3 \text{ defective DVD players from } 8) = \frac{\binom{4}{3}\binom{11}{5}}{\binom{15}{8}}$$ ***M1A1***

Note:	Award ***M1*** for an expression of this form containing three combinations.

$$= \frac{\frac{4!}{3!1!} \times \frac{11!}{5!6!}}{\frac{15!}{8!7!}}$$ ***M1***

$$= \frac{56}{195}$$ ***A1***

(b) $$\mathrm{P}(9^{\text{th}} \text{selected is } 4^{\text{th}} \text{defective player} \mid 3 \text{ defective in first } 8) = \frac{1}{7}$$ ***(A1)***

$$\mathrm{P}(9^{\text{th}} \text{selected is } 4^{\text{th}} \text{defective player}) = \frac{56}{195} \times \frac{1}{7}$$ ***M1***

$$= \frac{8}{195}$$ ***A1***

[7 marks]

https://xtremepape.rs/

10. (a) let the first three terms of the geometric sequence be given by u_1, $u_1 r$, $u_1 r^2$

$\therefore u_1 = a + 2d$, $u_1 r = a + 3d$ and $u_1 r^2 = a + 6d$ ***(M1)***

$\frac{a+6d}{a+3d} = \frac{a+3d}{a+2d}$ ***A1***

$a^2 + 8ad + 12d^2 = a^2 + 6ad + 9d^2$ ***A1***

$2a + 3d = 0$

$a = -\frac{3}{2}d$ ***AG***

(b) $u_1 = \frac{d}{2}$, $u_1 r = \frac{3d}{2}$, $\left(u_1 r^2 = \frac{9d}{2}\right)$ ***M1***

$r = 3$ ***A1***

geometric 4th term $u_1 r^3 = \frac{27d}{2}$ ***A1***

arithmetic 16th term $a + 15d = -\frac{3}{2}d + 15d$ ***M1***

$= \frac{27d}{2}$ ***A1***

Note: Accept alternative methods.

[8 marks]

https://xtremepape.rs/

SECTION B

11. (a) $\frac{dy}{dx} = 2x - \frac{1}{2}x^3$ ***A1***

$x\left(2 - \frac{1}{2}x^2\right) = 0$

$x = 0, \pm 2$

$\frac{dy}{dx} = 0$ at $\left(0, \frac{9}{8}\right), \left(-2, \frac{25}{8}\right), \left(2, \frac{25}{8}\right)$ ***A1A1A1***

Note: Award ***A2*** for all three x-values correct with errors/omissions in y-values.

[4 marks]

(b) at $x = 1$, gradient of tangent $= \frac{3}{2}$ ***(A1)***

Note: In the following, allow ***FT*** on incorrect gradient.

equation of tangent is $y - 2 = \frac{3}{2}(x-1)$ $\left(y = \frac{3}{2}x + \frac{1}{2}\right)$ ***(A1)***

meets x-axis when $y = 0$, $-2 = \frac{3}{2}(x-1)$ ***(M1)***

$x = -\frac{1}{3}$

coordinates of T are $\left(-\frac{1}{3}, 0\right)$ ***A1***

[4 marks]

(c) gradient of normal $= -\frac{2}{3}$ ***(A1)***

equation of normal is $y - 2 = -\frac{2}{3}(x-1)$ $\left(y = -\frac{2}{3}x + \frac{8}{3}\right)$ ***(M1)***

at $x = 0$, $y = \frac{8}{3}$ ***A1***

Note: In the following, allow FT on incorrect coordinates of T and N.

lengths of $\text{PN} = \sqrt{\frac{13}{9}}$, $\text{PT} = \sqrt{\frac{52}{9}}$ ***A1A1***

area of triangle PTN $= \frac{1}{2} \times \sqrt{\frac{13}{9}} \times \sqrt{\frac{52}{9}}$ ***M1***

$= \frac{13}{9}$ (or equivalent *e.g.* $\frac{\sqrt{676}}{18}$) ***A1***

[7 marks]

Total [15 marks]

https://xtremepape.rs/

12. (a) using the factor theorem $z+1$ is a factor *(M1)*
$z^3+1=(z+1)(z^2-z+1)$ ***A1***

[2 marks]

(b) (i) **METHOD 1**

$z^3=-1 \Rightarrow z^3+1=(z+1)(z^2-z+1)=0$ *(M1)*

solving $z^2-z+1=0$ ***M1***

$z=\frac{1\pm\sqrt{1-4}}{2}=\frac{1\pm i\sqrt{3}}{2}$ ***A1***

therefore one cube root of -1 is γ ***AG***

METHOD 2

$\gamma^2=\left(\frac{1+i\sqrt{3}}{2}\right)^2=\frac{-1+i\sqrt{3}}{2}$ ***M1A1***

$\gamma^3=\frac{-1+i\sqrt{3}}{2}\times\frac{1+i\sqrt{3}}{2}=\frac{-1-3}{4}$ ***A1***

$=-1$ ***AG***

METHOD 3

$\gamma=\frac{1+i\sqrt{3}}{2}=e^{i\frac{\pi}{3}}$ ***M1A1***

$\gamma^3=e^{i\pi}=-1$ ***A1***

(ii) **METHOD 1**

as γ is a root of $z^2-z+1=0$ then $\gamma^2-\gamma+1=0$ ***M1R1***

$\therefore \gamma^2=\gamma-1$ ***AG***

Note: Award ***M1*** for the use of $z^2-z+1=0$ in any way.
Award ***R1*** for a correct reasoned approach.

METHOD 2

$\gamma^2=\frac{-1+i\sqrt{3}}{2}$ ***M1***

$\gamma-1=\frac{1+i\sqrt{3}}{2}-1=\frac{-1+i\sqrt{3}}{2}$ ***A1***

continued …

https://xtremepape.rs/

Question 12 continued

(iii) **METHOD 1**

$(1-\gamma)^6 = (-\gamma^2)^6$ ***(M1)***

$= (\gamma)^{12}$ ***A1***

$= (\gamma^3)^4$ ***(M1)***

$= (-1)^4$

$= 1$ ***A1***

METHOD 2

$(1-\gamma)^6$

$= 1 - 6\gamma + 15\gamma^2 - 20\gamma^3 + 15\gamma^4 - 6\gamma^5 + \gamma^6$ ***M1A1***

Note:	Award ***M1*** for attempt at binomial expansion.

use of any previous result *e.g.* $= 1 - 6\gamma + 15\gamma^2 + 20 - 15\gamma + 6\gamma^2 + 1$ ***M1***

$=1$ ***A1***

Note:	As the question uses the word 'hence', other methods that do not use previous results are awarded no marks.

[9 marks]

continued …

https://xtremepape.rs/

Question 12 continued

(c) **METHOD 1**

$$\boldsymbol{A}^2 = \begin{pmatrix} \gamma & 1 \\ 0 & \frac{1}{\gamma} \end{pmatrix}\begin{pmatrix} \gamma & 1 \\ 0 & \frac{1}{\gamma} \end{pmatrix} = \begin{pmatrix} \gamma^2 & \gamma + \frac{1}{\gamma} \\ 0 & \frac{1}{\gamma^2} \end{pmatrix}$$ ***A1***

$$\boldsymbol{A}^2 - \boldsymbol{A} + \boldsymbol{I} = \begin{pmatrix} \gamma^2 - \gamma + 1 & \gamma + \frac{1}{\gamma} - 1 \\ 0 & \frac{1}{\gamma^2} - \frac{1}{\gamma} + 1 \end{pmatrix}$$ ***M1***

from part (b)

$\gamma^2 - \gamma + 1 = 0$

$\gamma + \frac{1}{\gamma} - 1 = \frac{1}{\gamma}(\gamma^2 - \gamma + 1) = 0$ ***A1***

$\frac{1}{\gamma^2} - \frac{1}{\gamma} + 1 = \frac{1}{\gamma^2}(\gamma^2 - \gamma + 1) = 0$ ***A1***

hence $\boldsymbol{A}^2 - \boldsymbol{A} + \boldsymbol{I} = \boldsymbol{0}$ ***AG***

METHOD 2

$$\boldsymbol{A}^2 = \begin{pmatrix} \frac{-1 + i\sqrt{3}}{2} & 1 \\ 0 & \frac{-1 - i\sqrt{3}}{2} \end{pmatrix}$$ ***A1A1A1***

Note: Award 1 mark for each of the non-zero elements expressed in this form.

verifying $\boldsymbol{A}^2 - \boldsymbol{A} + \boldsymbol{I} = \boldsymbol{0}$ ***M1AG***

[4 marks]

continued …

https://xtremepape.rs/

Question 12 continued

(d) (i) $\boldsymbol{A}^2 = \boldsymbol{A} - \boldsymbol{I}$

$\Rightarrow \boldsymbol{A}^3 = \boldsymbol{A}^2 - \boldsymbol{A}$ ***M1A1***

$= \boldsymbol{A} - \boldsymbol{I} - \boldsymbol{A}$ ***A1***

$= -\boldsymbol{I}$ ***AG***

Note: Allow other valid methods.

(ii) $\boldsymbol{I} = \boldsymbol{A} - \boldsymbol{A}^2$

$\boldsymbol{A}^{-1} = \boldsymbol{A}^{-1}\boldsymbol{A} - \boldsymbol{A}^{-1}\boldsymbol{A}^2$ ***M1A1***

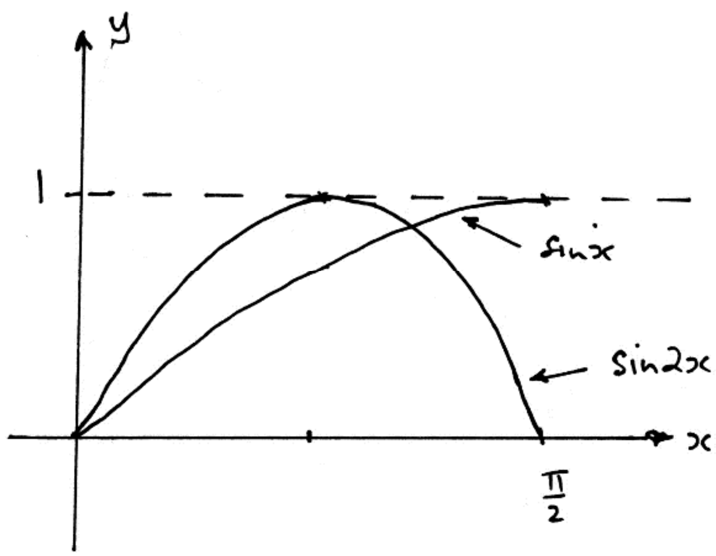
$\Rightarrow \boldsymbol{A}^{-1} = \boldsymbol{I} - \boldsymbol{A}$ ***AG***

Note: Allow other valid methods.

[5 marks]

Total [20 marks]

13. (a) (i)



A2

Note: Award ***A1*** for correct $\sin x$, ***A1*** for correct $\sin 2x$.

Note: Award ***A1A0*** for two correct shapes with $\frac{\pi}{2}$ and/or 1 missing.

Note: Condone graph outside the domain.

(ii) $\sin 2x = \sin x,\ 0 \le x \le \frac{\pi}{2}$

$2\sin x \cos x - \sin x = 0$ ***M1***

$\sin x(2\cos x - 1) = 0$

$x = 0, \frac{\pi}{3}$ ***A1A1*** ***N1N1***

continued …

https://xtremepape.rs/

Question 13 continued

(iii) $\text{area} = \int_0^{\frac{\pi}{3}} (\sin 2x - \sin x)\,\mathrm{d}x$ ***M1***

Note:	Award ***M1*** for an integral that contains limits, not necessarily correct, with $\sin x$ and $\sin 2x$ subtracted in either order.

$= \left[-\frac{1}{2}\cos 2x + \cos x\right]_0^{\frac{\pi}{3}}$ ***A1***

$= \left(-\frac{1}{2}\cos\frac{2\pi}{3} + \cos\frac{\pi}{3}\right) - \left(-\frac{1}{2}\cos 0 + \cos 0\right)$ ***(M1)***

$= \frac{3}{4} - \frac{1}{2}$

$= \frac{1}{4}$ ***A1***

[9 marks]

(b) $\int_0^1 \sqrt{\frac{x}{4-x}}\,\mathrm{d}x = \int_0^{\frac{\pi}{6}} \sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}} \times 8\sin\theta\cos\theta\,\mathrm{d}\theta$ ***M1A1A1***

Note:	Award ***M1*** for substitution and reasonable attempt at finding expression for $\mathrm{d}x$ in terms of $\mathrm{d}\theta$, first ***A1*** for correct limits, second ***A1*** for correct substitution for $\mathrm{d}x$.

$\int_0^{\frac{\pi}{6}} 8\sin^2\theta\,\mathrm{d}\theta$ ***A1***

$\int_0^{\frac{\pi}{6}} 4 - 4\cos 2\theta\,\mathrm{d}\theta$ ***M1***

$= [4\theta - 2\sin 2\theta]_0^{\frac{\pi}{6}}$ ***A1***

$= \left(\frac{2\pi}{3} - 2\sin\frac{\pi}{3}\right) - 0$ ***(M1)***

$= \frac{2\pi}{3} - \sqrt{3}$ ***A1***

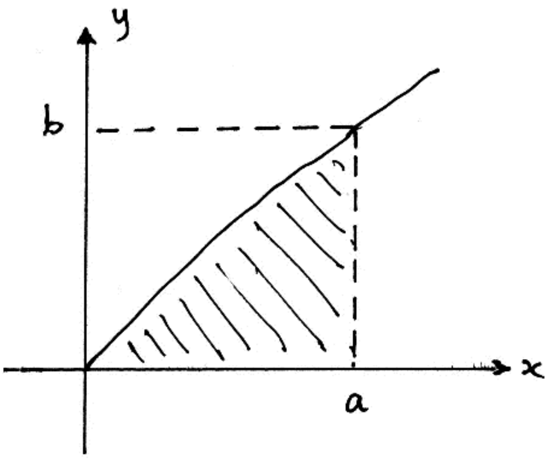
[8 marks]

continued …

https://xtremepape.rs/

Question 13 continued

(c) (i)



M1

from the diagram above

the shaded area $=\int_0^a f(x)\,\mathrm{d}x = ab - \int_0^b f^{-1}(y)\,\mathrm{d}y$ ***R1***

$= ab - \int_0^b f^{-1}(x)\,\mathrm{d}x$ ***AG***

(ii) $f(x)=\arcsin\frac{x}{4} \Rightarrow f^{-1}(x)=4\sin x$ ***A1***

$\int_0^2 \arcsin\left(\frac{x}{4}\right)\mathrm{d}x = \frac{\pi}{3} - \int_0^{\frac{\pi}{6}} 4\sin x\,\mathrm{d}x$ ***M1A1A1***

Note:	Award ***A1*** for the limit $\frac{\pi}{6}$ seen anywhere, ***A1*** for all else correct.

$= \frac{\pi}{3} - \left[-4\cos x\right]_0^{\frac{\pi}{6}}$ ***A1***

$= \frac{\pi}{3} - 4 + 2\sqrt{3}$ ***A1***

Note:	Award no marks for methods using integration by parts.

[8 marks]

Total [25 marks]

https://xtremepape.rs/